TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 522

THE ANALYSIS OF AIRCRAFT STRUCTURES AS SPACE FRAMEWORKS

METHOD BASED ON THE FORCES IN THE LONGITUDINAL MEMBERS

By Herbert Wagner

From Zeitschrift für Flugtechnik und Motorluftschiffahrt
August 14, 1928

Washington
July, 1929

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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Part I. Introduction

The following examples do not take up the discussion of viewpoints to be heeded in determining the design of a framework for given external conditions. Rather they are methods for determining the forces in airplane fuselages and wings, though similar considerations are applied to certain simple cases of a different kind. The object of this treatise is to summarize and amplify these considerations from definite viewpoints.

The static construction of fuselages and wings leads mostly to the use of frameworks which are to be regarded as space frameworks, but which are often rendered statically indeterminate by supplementary space diagonals. Almost all these airplane frameworks exhibit still another characteristic, which considerably simplifies their calculation under certain conditions, namely, the transverse frames are usually parallel. What follows will contain a method for the calculation of such stat-

*"Ueber räumliche Flugzeugfachwerke. Die Längsstabkraftmethode." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, August 14, 1928, pp. 337-347.

ically determinate space frameworks. A few principles will also be indicated for the calculation of statically indeterminate frameworks.

In order to make the subsequent illustrations easy to understand, the general principles of statics are repeated in Part II.

Part II. General Principles of Statically Determinate Space Frameworks

1. Number of Members

Between the number of members s and the number of joints k in a statically determinate space framework, there is the relation $s = 3k - 6$. This condition is not absolute, since there may exist a so-called exceptional case, which will not happen, however, in what follows. The required number of members is also not necessary. In Figure 2a, for example, the fact that the points III and IV have no directly connecting member does not matter, provided the member connecting the points II and V through III has the requisite buckling strength. From here on we will speak of such frameworks also as statically determinate.

2. Law of Superposition

If we know, in a framework, the forces S_1, which are produced in the members by a balanced group P_1 of external forces, and also the forces S_2 produced by a second group of forces P_2, the forces S, produced by the simultaneous action of both

groups P_1 and P_2, then equal $S_1 + S_2$.

3. "Simple" and "Complex" Frameworks

Simple frameworks are those whose forces can be determined directly by beginning with a joint where three members meet and by then continuing the determination with a second joint where only three more forces are unknown, and so on until all the forces are determined. The applicability of this method depends on the assumption of a certain "simple" type of framework (Föppl, "Graphische Statik" pages 15 and 176).

The framework shown in Figure 1a is "complex." Since more than three members meet at each joint, one cannot begin the determination at any one of the joints. In the framework shown in Figure 1b, the forces can indeed be first determined in the members represented by light lines (first taking joint I and then joint II), but the above-mentioned difficulties are encountered, in attempting to determine the forces in the remaining members, represented by the heavy lines, which constitute the so-called "base figure" of the framework.

The later examples concern the determination of the forces in such base figures. In this connection we shall make no use of the Henneberg method (Föppl, "Graphische Statik," p. 185).

4. Space Frameworks

Every surface framework consisting of triangles and enclosing a simple continuous space generally yields, if the edges are regarded as members and the corners as joints, a statically determinate framework which is called a space framework. No member passes through the interior of a space framework.

If many of the members of a space framework are joined in one plane, this plane framework, composed of nothing but triangles, is always statically determinate, when all the junctions are on the edges.

5. Changing the Diagonals

In the statically determinate framework subjected to the external forces P, as shown in Figure 2a, there are, for example, several members in its front surface which form a statically determinate plane framework in one plane. In such a case we distinguish between the edge members (1 to 6) of this surface and the diagonal members (7 to 9) or simply "diagonals."

The framework shown in Figure 2b is subjected to the same external forces and differs from the framework in Figure 2a only in the different (statically determinate) arrangement of the diagonals in the front field. It can then be shown that, despite the changed diagonals in this one plane, the same forces prevail in both frameworks in all the members not in this plane.

If, for example, we have calculated the forces in the frame-

work 2b, we can find the still unknown forces in the front field of the framework 2a by adding the forces in 2c to the forces in the front field of the framework 2b. Here D represents the diagonal force in the framework 2b.

6. Additional Forces

We shall often make use of the following line of reasoning. A statically determinate framework is subjected to a group of balanced external forces (the forces P in Figure 3). Several of these forces are supposed to lie in one plane of the framework (e.g., the forces P_1). We now apply in this plane the additional forces Z, and indeed at each joint always a pair of equal and opposite forces, so that the framework is subjected to no load due to these additional forces, and consequently the forces in its longitudinal members are not affected. These additional forces are so chosen that some of them balance the external forces lying in this plane. Thus, for example, the once-crossed additional forces Z balance the once-crossed forces P_1. Since the equilibrium of the whole framework remains unchanged, the remaining twice-crossed additional forces Z must of necessity balance the remaining twice-crossed external forces P_2. The once-crossed forces are therefore in equilibrium of themselves, as likewise the twice-crossed forces, and we can determine the forces for each group separately. The final forces are found by means of the law of superposition (Section 2).

7. Six-Sided Frameworks

Before passing to the method which is the special subject of this treatise, I would like to indicate briefly the usual method of determining the forces in a space framework, all the members of which lie in six planes (Fig. 4a). Such a framework is generally complex. In order to calculate the forces, the diagonals have to be changed to simple ones. In fact, they must be arranged so they will radiate from only four joints (Fig. 4b). We thus obtain a simple framework, in which the forces can be determined directly. It is necessary, however, to proceed with the determination only until one diagonal force is known. In the field of this diagonal force, the forces in the actual diagonals are then determined according to Section 5. After these forces are known in one field, the forces in all the other members can be determined by the usual method of the resolution of forces.

8. Forces Exerted on the Middle Joint of the Framework

Let the force P_1 (Fig. 5) be one of the forces to which the framework is subjected. It lies, e.g., in the plane of the right front surface. Hence the diagonals cannot be changed directly according to Section 7, since, in using this method, the joint to which P_1 is applied would be suppressed (Fig. 4b). Additional forces Z are therefore applied to any two joints on the front surface, which are also corner points of the frame-

work, in such a way that two of them (namely, the once-crossed ones in Fig. 5) balance the force P_1. We now determine the forces in the members separately, first for the twice-crossed forces (e.g., according to Section 7), and secondly, for the once-crossed forces, to which only the members in the front surface of the framework are subjected. The final forces in the members are obtained by the superposition of the two forces in the members.

9. Quasi Statically Determinate Space Frameworks

The five members of the right front surface of the space framework (Fig. 6) form, as always, a statically determinate plane framework. The space framework is subjected to the external forces P. Let the member forces be determined. If the two oppositely directed forces Q in the front plane of the framework are added, only the members lying in this plane receive an additional load through these two forces, while the forces in all members not in this plane remain unchanged, however great may be the force Q and the deformation produced by it. These forces Q, however, may come just as well from a second diagonal member in the front plane. Thereby the front field would indeed become statically indeterminate and the forces in these members would change. The members not lying in this plane, however, would remain unchanged.

Attention should be called, however, to the fact that it

is not a question here of diagonals which, in the double diagonal bracing of the field, consist of purely tensile members under no or only a very little initial tension. One of the diagonals is then without tension and has no effect on the member forces.

On the other hand, it should be noted that every diagonal force passing through the inside of a space framework (e.g., from I to II in Fig. 6) stresses all the members (aside from exceptional cases) and that, therefore, such a diagonal would then transform the framework into a statically indeterminate space framework in the full sense of the term. This reasoning can be carried further and be expressed in the following theorem.

Theorem.- If, in a space framework bounded by plane surfaces, further members are added in these surfaces, so that these surfaces form statically indeterminate plane frameworks, we then obtain a quasi statically determinate space framework, in which we can calculate all members not lying in these planes just the same as in the original statically determinate framework, the static indeterminateness affecting only the abovementioned surfaces.

It should be pointed out that statically indeterminate surfaces, which have no common edge members, do not affect one another, i.e., each can be calculated by itself as statically indeterminate. If, on the contrary, they have edge members in common, they then mutually affect one another, i.e., changing the dimensions of one member in one of these surfaces also af-

fects the forces in the members in the other surfaces.

The following considerations (as well as the ones in Section 7), for the determination of the forces in the members in statically determinate space framework, can also be applied directly to quasi statically determined space frameworks, in so far as it concerns the determination of the forces in members which do not lie in statically indeterminate surfaces. The forces in the members even in the statically indeterminate portions of the framework can then be easily determined.

Part III. Method of Analysis Based on the Forces in the Longitudinal Members of a Space Framework with Straight Members

All the examples in Parts III and IV refer to complex airplane space frameworks with parallel transverse frames. In contrast with the method given in Section 7, the following method is also applicable to frameworks with more than six faces. Its application requires some practice, but in many cases it gives simple numerical results. It also gives, from the beginning, a good idea of the course of the forces.

In complex frameworks it is always necessary to determine the force in at least one member. The next task then consists in finding the forces in the other members. Sections 11, 13 and 14 show how the diagonal forces are found in the transverse frames, while Section 12 gives the method for finding the forces in the other members.

10. Notation

External forces are designated by P and Q, their size and location being indicated by subscripts. Only such transverse frames as have diagonals are called frames in the following and they are designated by a, b, etc. The frame members, their lengths and also the other frame dimensions are designated by the frame letters and a subscript: the edge members, for example, by a_1, b_3, etc.; the frame diagonals, for example, by a_d, a_e, the general designations for frame members being a_m, b_m. Forces in frame members are designated by A and B and the subscript number or letter of the corresponding member, for example, A_d. The perpendicular distance between two frames is designated by l_{ab}, l_{bc}, etc.

All members of the structure which do not belong to the frames are called envelope members. The envelope members which connect the transverse frames and form the edges of the space framework are called chord members or spars, the other members forming the envelope bracing. Bracing members extending around the framework in a single plane form an intermediate transverse frame. The other members of the envelope bracing are called envelope diagonals. The intermediate frames are not necessarily parallel to the main frames. Their position does not affect the following discussion.

The envelope members and their lengths are designated by s and the member number as the subscript, for example, s_3, or in general by s_n. The forces in the envelope members are designated by S with subscript. Projections of members, forces, etc., are designated by an accent, for example, S_2'.

11. Torsion, General Case

All the external forces acting on the space framework are considered to lie in the plane of the two transverse-frame surfaces. It follows from the consideration of the balancing of the external forces, that this is possible only when these external forces, as resultants (naturally opposite on both surfaces), produce a like moment. This torsional moment is designated by M.

In the general case the two parallel frames may take the form of any desired polygons (Fig. 7a). Then all the envelope members in Figure 7a are chord members or spars, there being no envelope bracing members represented. If there were any such bracing members, their forces would obviously all be zero, provided the middle joint of the envelope were not subjected to any external force (Section 8).

Since, for example, the external force acting on the joint I lies, by assumption, in the plane of the frame a, it follows from the equilibrium of the components perpendicular to this plane that the force of the only envelope member proceeding

from this joint is zero. Hence we can consider as omitted this and all other envelope members which meet a corner of a transverse frame singly.

Of the remaining envelope members, therefore, there will always be two which meet in a frame corner, and which connect the two frames in a zigzag. If we form at a joint the sum of the components perpendicular to the plane of the transverse frame, we find that these components must be equal and opposite in the two members. However, since the remaining envelope members form a zigzag, we find that the values of the components of the forces in the envelope members perpendicular to the plane of the frame are equal in all the members, one member being under tension, the next under compression, etc. This component, which is constant for all envelope members, is termed the longitudinal component of force in the envelope members or "longitudinal member force" for short. It is designated by L_{ab}.

The line connecting the end points of the two forces in the envelope members radiating from a joint (Fig. 7a) is parallel to the plane of the transverse frame. It follows that this triangle of forces is geometrically similar to the triangle III-II-IV, which consists of both envelope members and the line b_d, connecting the joints of the other transverse frame b, from which these envelope members proceed. The ratio of the forces to the corresponding lengths of the members is $L_{ab} : l_{ab}$. This ratio, which will often be used, is designated

by μ_{ab} and is termed the μ-value. Hence, if we know the longitudinal member force, we can calculate the forces in the envelope members from the lengths of the members with the aid of the formula, for example,

$$S_1 = \mu_{ab}\, s_1 \quad \text{or, in general,} \quad S_n = \mu_{ab}\, s_n.$$

The resultant force A_{bd} of the two envelope forces S_1 and S_2, acting on the frame a, lies in the plane of the frame a. It is parallel to the line b_d connecting the end points III and IV of the other frame b and (as follows from the similarity of the triangle of forces to the triangle of members) proportional to the length of this connecting line, that is, for example,

$$A_{bd} = \mu_{ab}\, b_d \quad \text{or, in general,} \quad A_{bm} = \mu_{ab}\, b_m.$$

Very similar relations naturally obtain for the corner forces acting on the frame b:

$$B_{am} = \mu_{ab}\, a_m.$$

These "corner forces" are the forces exerted by the envelope on the transverse frame at the corners of the frames. The resultant of all the corner forces acting on a frame must balance the torsional moment.

We now project the whole framework in Figure 7a on a plane parallel to the transverse frames (Fig. 7b), omitting the unstressed envelope members. The projection S_n' of the forces

in the envelope members also is to the length s_n' of the projection of the corresponding members as $L_{ab} : l_{ab} = \mu_{ab}$. We now calculate the moment of all the forces in the envelope members about any point O and, with the designations of Figure 7b, obtain

$$M = \sum^n r_n S_n' = \sum^n r_n \frac{L_{ab}}{l_{ab}} s_n' = \frac{L_{ab}}{l_{ab}} 2 F_{ab}$$

Here F_{ab} is the hatched area in Figure 7b, which is enclosed in the projection of the envelope members. From this equation we obtain the μ value

$$\mu_{ab} = \frac{M}{2 F_{ab}} .$$

If we have to calculate the torsion of such a space framework, we must first determine the value F_{ab} of the hatched area. From this equation we then calculate μ_{ab} and finally obtain the desired forces in the members:

$$L_{ab} = \mu_{ab} \; l_{ab} \qquad S_n = \mu_{ab} \; s_n$$

$$A_{bm} = \mu_{ab} \; b_m \qquad B_{am} = \mu_{ab} \; a_m$$

We now obtain the forces in the members of both transverse frames from a Crémona plane, by applying to the frame, as shown in Figure 7c for frame a, the external forces and also the corner forces (parallel to the members or, as the case may be, to the geometric diagonals of the other frame). Since the magnitude of the hatched area is independent of the distance between the two frames, the corner forces and the torsional forces

in the members of the frame are also independent of the distance L_{ab} between the frames. The following theorem can be easily demonstrated.

Theorem.- The hatched area and hence also the forces in the members of the parallel transverse frames of a framework under torsion, do not change when the two transverse frames are moved relatively to each other, but not rotated. If, for example, the torsional moments act in like manner on a right prism and on an oblique prism which have like transverse frames, the forces in the members of the frames are the same in both cases.

12. Determination of the Forces in the Envelope Members

In a space framework (Fig. 8a) under torsion, let the four envelope surfaces be flat. In order to analyze this framework, we imagine the surface bracing transformed into simple diagonals, as indicated in one field in Figure 8a. This does not change the diagonal forces in the transverse frames (Section 5). In determining F_{ab} (Fig. 8b), these imaginary envelope diagonals, directly connecting the transverse frames must, of course, be drawn. The transverse-frame diagonals (indicated in Fig. 8c for the frame a) are then calculated according to Section 11 with the aid of the corner forces.

Various methods may be employed to determine the remaining forces in the members, one or the other being preferred according to circumstances. For example, we can determine these forces

from joint to joint as in a simple framework, since we already know the forces in two members, namely, the transverse-frame diagonals. Or we can actually determine all the forces in the members of the imaginary simple envelope bracing, according to Section 11, and then determine, according to Section 5, the forces in the members with changed envelope bracing.

The following method is generally the simplest. We determine for both frames (Fig. 8c) not only the forces in the diagonals of the frame, but also all the other forces in the members of the frame (Cremona planes). We then draw all envelope surfaces (Fig. 8d) as plane frameworks. (If the lateral envelope surfaces are not inclined greatly toward the vertical ones, we can generally employ without great error the lateral outline of the space framework for this purpose.) The forces exerted by the transverse frames on these plane frameworks are now the already known corner forces A_{bm} and B_{am}. They must now be applied, however in the opposite direction and to the same joints to which they were applied in Figure 8c. In order to obtain external equilibrium of the forces for the individual envelope surfaces, we must apply, e.g., additional forces at the joints I to IV in the direction of the spars. The additional force Z_{II} represents, for example, the forces exerted at the joints VI and VII by the front envelope surface on the upper envelope surface (and vice versa). These additional forces, which balance the corner forces on every envelope surface, must be of

like magnitude at the corresponding corner points of every envelope surface (e.g., Z_{II} at the point II on the lateral surface and $-Z_{II}$ on the upper surface). Cremona planes are now drawn also for the envelope surfaces. The forces thus obtained in the bracing members are final. The final forces in the spars are obtained by superposition of the corresponding forces in the members of both force planes in which they occur.

Likewise we obtain the final forces in the edge members of the transverse frames through superposition of the forces produced in the force plane of the transverse frame and of the envelope surface (whereby one of the two is generally zero or the final force in the member is zero).

If, e.g., the members a_3 and b_3 are not exactly parallel, so that the envelope surface is distorted and the envelope members of this surface no longer lie in one plane, the bracing cannot be changed and the method of Section 11 cannot be used (no more than the method of Section 7). Nevertheless, if the distortion of the surfaces is only slight, the error resulting from altering the bracing in the calculation of the forces in the frame diagonals is then so small that it can be disregarded.

13. Torsion, Special Cases

<u>1. Trapezoidal frames</u>.- Let the transverse-frame edge members in the same envelope surface be parallel, so that the four envelope surfaces are plane (Fig. 9a). For the hatched surface

we obtain the simple expression

$$2\ F_{ab} = a_h\ b_1 + b_h\ a_2 =$$

$$= a_h\ b_2\ b_h\ a_1 .$$

This hatched surface does not change, of course, when the envelope diagonals shown in Figure 9a are replaced by others. If both transverse frames are rectangular, we obtain, with the notation of Figure 9b

$$2\ F_{ab} = a_1\ b_2 + a_2\ b_1 .$$

2. Space framework having the form of the frustrum of a pyramid with bases or transverse frames of any shape.- In this case the perimeters of the two frames are geometrically similar. Let the ratio of the corresponding sides be $b_m : a_m = \lambda$. If we designate the area of the frame a by F_a, it is easy to show that

$$2\ F_{ab} = 2\ \lambda\ F_a .$$

For the corner forces acting on the frame a, we obtaim

$$A_{bm} = \frac{M}{2F_{ab}}\ b_m = \frac{M}{2\lambda F_a}\ \lambda\ a_m = \frac{M}{2F_a}\ a_m .$$

In this case the corner forces are independent of the size of the other frame and are proportional to the length of the side of the transverse frame on which they act and whose direction they have.

In particular we obtain for the frustrum of a pyramid with rectangular bases and with the sides a_1, a_2, and b_1, b_2,

for example, for frame a,

$$A_{b_1} = \frac{M}{2\ a_1\ a_2}\ a_1 = \frac{M}{2\ a_2}$$

$$A_{b_2} = \frac{M}{2\ a_1},$$

so that the moment of the two forces A_{b_1} (Fig. 9b) becomes $A_{b_1}\ a_2 = \frac{M}{2}$, likewise the moment of the two forces A_{b_2}. In a space framework having the form of the frustrum of a pyramid with rectangular bases, the torsional moment is distributed equally between the two opposite envelope faces. It should be noted, however, that this applies only to geometrically similar transverse frames or bases.

14. Longitudinal and Transverse Forces

In what follows, we shall not understand by longitudinal forces the forces in the longitudinal members, but external forces acting perpendicularly to the transverse frames. By a few simple examples we will now show how, with the aid of additional forces, the action of longitudinal and transverse forces is reduced to the action of a torsional moment.

If, for example, four mutually balanced longitudinal forces (P_1 and P_2) act on the space framework at the transverse frame b (Fig. 10), the diagonal force A_d in the opposite frame a is very easily calculated. We apply to the frame a four additional forces Z_1 of the magnitude

$$Z_1 = \frac{P_2\ b_2}{l_{ab}} = \frac{P_1\ b_1}{l_{ab}}\ .$$

Likewise we imagine additional forces applied to the frame b inside its plane in such manner that, for example, the forces P_1, together with a portion of these additional forces, balance the once-crossed Z_1 on the upper envelope surface and likewise the forces P_2 with their additional forces balance the force Z_1 on the lower envelope surface. The two groups of forces act only on the upper and lower envelope surfaces, so that no account needs to be taken of them in calculating the desired diagonal force in frame a.

The remaining additional forces in frame b must then balance the two remaining uncrossed additional forces Z_1 in the frame a, which produce a torsional moment. Hence we need not calculate the magnitude and location of the additional forces in the frame b, but only the forces Z_1 in the frame a and the loading of the desired diagonals due to the torsional moment $Z_1 a_h$ of the uncrossed forces Z_1, corresponding to the statements in the preceding sections. The remaining forces in the members can then be determined according to Section 12, whereby the corner forces for the rear frame in this case are setermined from the equilibrium of the forces on the plane envelope frameworks (Fig. 8d).

If, for example, the transverse frame a (Fig. 11a) is acted on by a transverse force Q, which is balanced in the frame b by the transverse forces P and the longitudinal forces P_L, and if, in particular, the longitudinal forces act

in the same envelope surfaces as the transverse force Q, it is easily seen that only the members of the upper envelope surface and of the frame b are stressed. All the other members are free from stress, in particular, the diagonal a_d.

If, however, the longitudinal forces P_{L_2} (Fig. 11b) also act on the lower side of the frame b, we then apply to the upper side of the frame b additional longitudinal forces Z_L of such magnitude that two of them are in external equilibrium with the longitudinal forces on the lower side.

$$Z_L = \frac{P_{L_2}\ b_2}{b_1} .$$

The once-crossed forces in Figure 11b do not now act on the unknown diagonal a_d of the frame a, the loading of this diagonal by the twice-crossed longitudinal forces being calculated as already described. The loading of the diagonal in frame a, due to the transverse force acting on this transverse frame, therefore depends only on the way the balancing longitudinal forces act on the other frame b. It does not depend on the way the transverse forces act on the frame b. All other members in Figures 10 and 11 can be calculated according to Section 12.

For subsequent uses in trapezoidal frames with plane envelope surfaces and in the case of longitudinal forces occurring singly (Fig. 10), both the frame diagonal forces are given in analytical form, using the notation of Figures 9a and 10.

$$A_d = + \bar{P}_1 \frac{a_d}{l_{ab}} \frac{b_h b_1}{2F_{ab}} \tag{1}$$

$$B_d = - P_1 \frac{a_d}{l_{ab}} \left(1 - \frac{a_h a_2}{2F_{ab}} + \frac{f_{ab}}{b_h} \frac{b_1 - b_2}{b_2}\right) \tag{2}$$

B_d depends therefore on the relative positions of the transverse frames (that is, on f_{ab}). In order to avoid misunderstanding, special attention is called to the fact that f_{ab}, in the projection perpendicular to the frames, indicates the height of the member a_2 above the member b_2.

In particular, if the transverse frames are rectangular, we obtain, with the notation of Figure 9b (F_a and F_b being the frame areas), the simple expressions

$$A_d = + \bar{P}_1 \frac{a_d}{l_{ab}} \frac{F_b}{2F_{ab}} \tag{1a}$$

$$B_d = - P_1 \frac{b_d}{l_{ab}} \left(1 - \frac{F_a}{2F_{ab}}\right) \tag{2a}$$

These equations for A_d and B_d hold good only for positions of the frame diagonals indicated in Figure 10. If a diagonal lies between the other frame corner points, then the sign is simply changed in the equation for the corresponding diagonal force.

15. A Few Applications

In the following illustrations all plane surfaces are to be considered as statically determinately braced. Nevertheless, only such bracing members are represented as are of especial interest.

Example 1.- The space framework in Figure 12, which represents the central portion of an airplane fuselage, is acted on at the top by the longitudinal forces P_1 of the wing and at the bottom by the longitudinal forces P_2 of the fuselage produced by the rudder loading. The two pentagonal surfaces (lying in the side walls of the fuselage) are here considered as transverse-frame surfaces and the others as envelope surfaces. To the upper envelope surface we apply the additional forces

$$Z = \frac{P_1 \; a_1}{l}.$$

The once-crossed forces act only on the upper envelope surface, while the uncrossed forces lie in the plane of the transverse frames and exert a torsional moment on the framework. The forces in the members are calculated according to Section 13, case 3. For the upper envelope surface the forces in the members are found by the law of superposition.

Example 2.- On the end section of a fuselage (Fig. 13) subjected to the transverse force Q, equilibrium is maintained in the transverse frame by the external longitudinal forces P_L and a number of forces P lying in the plane of this frame. In order to calculate the diagonal forces in the frame a, additional transverse forces $Z_Q = Q$ are applied. The diagonal forces in the frame a are then subjected only to the torsional moment $Q\ x$ of the once-crossed forces. They can be calculated according to Section 11.

None of the other forces acts on these diagonals. The twice-crossed force Z_Q can be resolved into two components in the planes of the adjoining envelope surfaces, and it then balances both moments of the longitudinal forces P_L and Z_L, whereby, along with the prevailing transverse forces, we must imagine additional forces applied to the frame b.

IV. Method of Analysis Based on Forces in the Longitudinal Members of a Statically Determinate Space Framework with Bent Chord Members

16. Forces in the Direction of an Imaginary Diagonal in the Middle Transverse Frame

By way of exception, we will also call the middle cross section b (Fig. 14) a transverse frame, although it has no diagonal braces, and will designate its members accordingly.

The illustrations in Section 17 represent primarily the fuselages of passenger airplanes in which, for a long distance, there is no transverse frame with diagonals (in order to leave the space as free as possible for the cabin) and in which, under certain conditions, account must be taken of the variations of the forces in the members, as compared with their values according to the previous sections, due to the bending of the chord members or spars. Instead of the actual gradual curvature of these spars, we can assume a sharp bend at a suitably chosen

place (Fig. 14) and regard the spars as otherwise straight. The considerations enumerated in this section furnish the foundation for those in Section 17.

The examples in this section can also be used for calculating the forces in the members of a statically indeterminate airplane framework (e.g., fuselage or wing) when the diagonal of the middle frame passing through the space inside the framework is chosen as a redundant member, the remaining statically determinate main framework corresponding in general to Figure 14. In order to calculate the statically indeterminate framework according to Maxwell-Mohr, we must first determine the effect of a force in the redundant member on the main framework, for which purpose the considerations of this section can be used. Such statically indeterminate frameworks can be calculated more easily, however, according to the method described in Sections 18-20.

The framework in Figure 14 is separated in Figures 15a and 15b into a right and a left part. In these two frameworks the frame b has no diagonals. These frameworks are therefore geometrically under-determinate. If we cause additional forces Bd_a to act on the framework 15a in the direction of the missing diagonal of frame b, the framework cannot then absorb these forces. Only when we simultaneously apply to frame b the longitudinal forces P_1 and P_2 of given magnitude, can the framework members receive the load Bd_a. The magnitude of the forces P_1 is found from equation (2) in Section 15, if we substitute

B_{d_a} for B_d and designate the length of the imaginary diagonal by b_d.

In framework 15b the forces B_{d_c} are applied in the direction of the missing diagonal of the frame b. In fact, B_{d_c} is made just great enough so that the added forces P_1 and P_2 (which are calculated according to equation (2) in Section 15 by substituting c for a) are equal and opposite to the corresponding forces in Figure 15a.

If we now fit the frameworks 15a and 15b together so as to form the framework in Figure 14, we can eliminate the longitudinal forces P_1 and P_2. The external loading B_d of the framework is the sum of B_{d_a} and B_{d_c}. The longitudinal forces P_1 are found from the equation

$$P_1 = \mp \frac{B_d}{\frac{B_{d_a}}{P_1} + \frac{B_{d_c}}{P_1}} .$$

The force in the diagonal of the frame a is now found from equation (1) in Section 16. For rectangular frames we can write the resultant diagonal force

$$A_d = \mp B_d \frac{a_d}{b_d} \frac{\frac{F_b}{2F_{ab}}}{\left(1 - \frac{F_a}{2F_{ab}}\right) + \frac{l_{ab}}{l_{bc}} \left(1 - \frac{F_c}{2F_{bc}}\right)} .$$

The negative sign applies to the case when the diagonal in the frame a passes out from the same spars as the imaginary diagonal in frame b. Otherwise the positive sign is used. The other forces in the members are now found according to Section 14.

17. Torsion

If a torsional moment M acts on the end frames of such a framework (Fig. 16a), the forces in the members are determined as follows. Two pairs of equal and opposite additional forces are applied to the middle unbraced frame b in the direction of an imaginary diagonal. In accordance with the preceding sections, we will designate these forces as $\pm B_d$. Their magnitude is such that, with the simultaneous action of the torsional moment and the additional forces $- B_d$, the force in the longitudinal member between the frames a and d is a constant L_{ab}, and likewise the force in the longitudinal member between the frames b and c is a constant L_{bc}. The magnitude of $- B_d$ is found, therefore, by applying to the frame b (Fig. 16b), according to Section 11, the corner forces $B_{am} = \mu_{ab}\ a_m$ produced by the forces in the envelope member between frame a and b, and also the corner forces $B_{cm} = \mu_{bc}\ c_m$ produced by the forces in the envelope member between frame b and c.

The forces in the frame diagonals A_d and C_d are first determined from the superposition of the action of the once-crossed forces (by applying, according to Section 11, the corner forces $A_{bm} = \mu_{ab}\ b_m$ to the frame a and likewise to the frame c) and of the uncrossed forces (e.g., for rectangular frames according to equation (2), Section 16). The forces in all the other members can now be determined according to Section 12.

V. Method of Analysis Based on Forces in the Longitudinal Members of a Statically Indeterminate Space Framework

All the following examples concern statically indeterminate airplane frameworks with parallel transverse frames.

18. Torsion. Assumption of a Constant Longitudinal Component of Force in the Members

The determination of the forces in the members of a statically indeterminate framework resolves itself into two parts, namely, the calculation of the actual forces in the members for the preliminary design of the members and the statically indeterminate calculation. If the forces in the members are poorly estimated, the calculations have to be repeated until a satisfactory agreement is obtained between the dimensions and stresses. The calculation requires great skill, especially when, as in airplane construction, the framework has to be designed for different cases of loading.

In airplane frameworks subjected to torsional moments (fuselage and wings), it is advantageous to proceed as follows. The forces in all frame diagonals are regarded as forces to be estimated. It is assumed at first that the frame-diagonal forces correspond to constant longitudinal components of forces in all envelope members between every two transverse frames (whereby

diagonals running from frame to frame are to be assumed as envelope bracing, even when the actual envelope bracing is different).* Corner forces, as above indicated, are therefore applied to every frame in addition to the external forces acting on it, e.g., to frame b, which lies between the frames a and c, the corner forces $B_{am} = \mu_{ab}\, a_m$ and $B_{cm} = \mu_{bc}\, c_m$ (Fig. 16b). For these balancing forces, we calculate all frame-diagonal forces and then also all other forces of the framework. With envelope diagonals running from frame to frame, we find the forces in the envelope members as $S_n = \mu_{ab}\, s_n$, etc. Otherwise, we find them as represented in Figure 8d.

The forces thus obtained now represent mostly very good approximations of the forces actually occurring in the members of the statically indeterminate framework, so that they form a very good basis for provisional design, or may often be regarded, even without any statically indeterminate calculation, as the final forces in the members.

If we wish to make an exact statically indeterminate calculation, we select, for example, in the framework of Figure 17, as redundant members, the diagonals of an end frame and those of the next frame (e.g., the diagonals of the frames c and d), so that there remains, as the main framework, the framework which embraces all the envelope members, the edge members of all the frames and the diagonal members of two neighboring frames. This

*This represents the extension of an assumption, which was made long ago by L. Staiger for simple cases.

main framework is not a simple space framework, though the base figure (consisting of the members of both frames a and b and the intervening envelope members) has straight spars and can therefore be easily calculated with the aid of the methods described in Sections 11 and 14.*

The usual procedure in making a statically indeterminate calculation is first to calculate the forces T in the members resulting from the action of the external forces, whereby the forces are assumed to be zero in all redundant members, and then to determine the various forces u_x in the members produced by the action of the forces 1 in the redundant members x. By this method the forces T in the members of the frameworks considered here are very large, and we obtain, according to the Maxwell-Mohr method, the final forces in the members as the difference between two very large members and must therefore calculate to many decimal places in order to attain sufficient accuracy.

The following method is preferable. We determine the forces T on the members under the action of the external loads with the simultaneous action of the forces in the redundant diagonals, as they were calculated under the assumption of constant

*Other diagonals might also be regarded as redundant members. They must be so situated, however, that the base figure of the main framework embraces the diagonals of two neighboring transverse frames. If, on the contrary, all the space diagonals of the intermediate frames were to be regarded as redundant members, the forces in the members would have to be found with the aid of the complicated method of Section 16. With many redundant members and with frequently bent spars, we would not attain our goal without still more complicated methods.

longitudinal forces for the initial design of the members. The forces u_x in the members and then also the final forces in the members are determined in the usual manner.* It is not then necessary to recalculate the forces T in the members, because they are already known from the initial design. Moreover the forces T already closely approximate the final forces in the members which, according to the additional forces obtained by the Maxwell-Mohr method, are only small, and we can therefore be satisfied with considerably fewer decimal places.

19. Basis and Field of Application

No generally applicable proof for the approximate correctness of the assumption of constant longitudinal force can, of course, be given. Hence I will only briefly describe the basis and call attention to the following line of reasoning.**

Under the assumption of constant longitudinal member forces in the envelope members between every two transverse frames, the torsional moment in each framework member is transmitted just the same as though only this member were present. We now consid-

*Whereby it must, of course, be noted that, contrary to the usual method of calculation, the forces T differ from zero in the redundant members.

**As the basis we can also consider the work of deformation of the framework. We then proceed from the fact that correct forces in the members correspond to the minimum of the total work of deformation of the framework. Since the envelope members absorb most of the work of deformation, their work of deformation must be as small as possible. It can then be shown that this is the case only when the longitudinal member force in all members between two transverse frames does not differ too much from a constant. This rather complicated basis, which is naturally not very exact, I would like to pass over in the interest of brevity.

er the framework resolved into separate statically determinate space frameworks, consisting of the members of the two neighboring transverse frames and the envelope members lying between them, so that all frames, with the exception of the end frames, naturally occur twice. We then let the corresponding torsional moment act on each of these space frameworks. Among other things, a displacement of the joints perpendicular to the plane of the transverse frame is connected with this torsional loading. The less these displacements of the joints of one and the same frame differ in two neighboring frameworks, the less will be the influence of both these framework parts in the actual statically indeterminate framework, and the more accurate will be the assumption of a constant longitudinal member force.

This consideration enables the deduction of the following viewpoints. The assumption of a constant longitudinal member force is more accurate: the greater the distance between the frames; the more uniform the distances between frames; the more similar the shape of the neighboring frames; the more uniform the action of the external torsional moment on the individual frames; and the more uniform the change in the dimensions of the individual spars and envelope diagonals. The dimensions of the frame members and especially of the frame diagonals has but little effect on the magnitude of the frame-diagonal forces.

In airplane construction the relations in the central section of the fuselage and wings is generally so favorable that

the error is very small in the members of the envelope surfaces including the edge members of the frames, the numerical calculations giving errors for the most part considerably less than 10%. If individual frame diagonals are only very lightly loaded and especially if no external torsional moment acts on a frame, the relative error for these members is naturally greater. Such lightly loaded members are being continually made weaker (if not entirely eliminated).

On the other hand, the error in the estimate in the vicinity of the frames which are suddenly exposed to very great torsional moments, especially when the direction of the torsional moment is changed and the transverse frames deviate greatly from the square shape, is very large, often as much as 50%. This is the case, e.g., at the junction of the fuselage and wing and in braced wings at the ends of the struts. Even in fuselage frameworks in which the envelope bracing is wanting in one field due to the removal of a portion of the upper envelope over the pilot's seat, great deviations from the estimate occur in this vicinity. In such cases an estimate on the basis of careful considerations and also a statically indeterminate calculation are necessary. Even here the statically indeterminate calculation for the whole wing framework can generally be dispensed with. It generally suffices to consider as cut away the portion of the framework between the two transverse frames which lie on both sides of the frame in which the disturbance occurs (i.e.,

three frames in all and the envelope members lying between them); to apply the forces in the members outwardly adjoining this portion of the framework (according to the above-indicated estimate) and the other forces as external loading; and to make the statically indeterminate calculation for this portion. The disturbance produced by the sudden change in the moment disappears rapidly when the frames are not too near together. If it is suspected, however, that the effect of this disturbance extends further, the statically indeterminate calculation is extended to the portion of the framework which lies between the next two frames, that is, to five frames in all (thrice statically indeterminate).

Of course in every individual case it must be left to the judgment and experience of the designer as to how far he trusts such an estimate and what other viewpoints he shall employ for the calculation or final design.

20. Torsion and Bending

Airplane frameworks are generally subjected simultaneously to torsional and bending stresses. The forces in the members will then be calculated as independently as possible for bending and torsion, so that various cases of loading can then be determined by the law of superposition. The separate calculation of the forces in the members for torsion and bending is always quite possible in wing frameworks, where, after finding the

center of shear of the separate cross sections, the external loading can be resolved into shearing forces, passing through the center of shear and into the residual torsional moment.

There are difficulties, however, especially in estimating the forces produced in the members by horizontal transverse forces in the fuselage (rudder loading), since the course of these transverse forces and of the corresponding bending moments depends largely on the manner of their transmission to the wing. In small airplanes this calculation is made more difficult by the irregularities produced by the cutaway for the pilot's seat. It is difficult to generalize concerning the course of these forces, because the number of different possibilities is very large, but the statically indeterminate calculation corresponding to the examples in Sections 18 and 19 present no difficulties.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics .

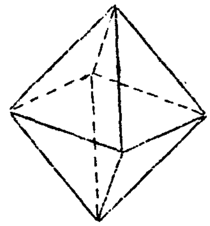


Fig.1a

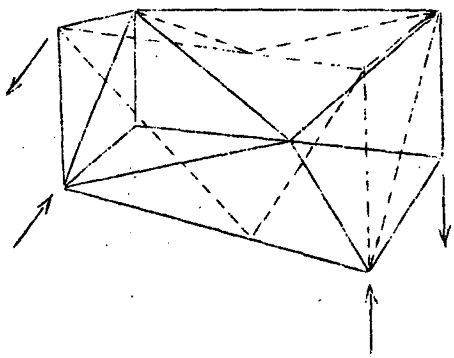


Fig.4a

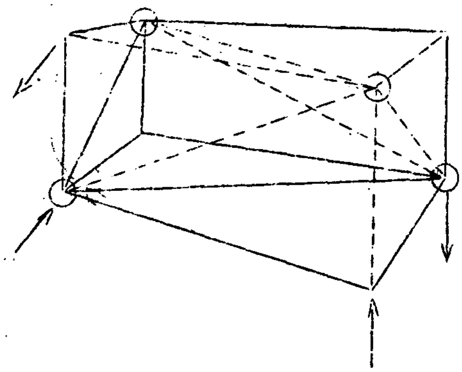


Fig.4b

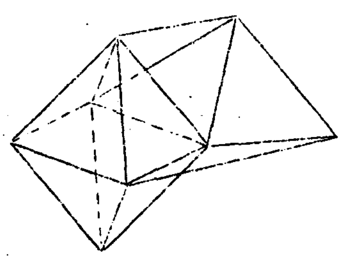


Fig.1b

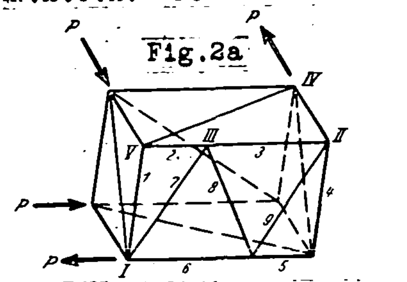


Fig.2a

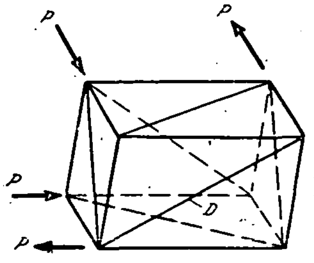


Fig.2b

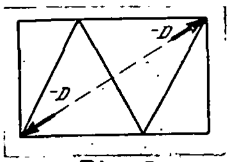


Fig.2c

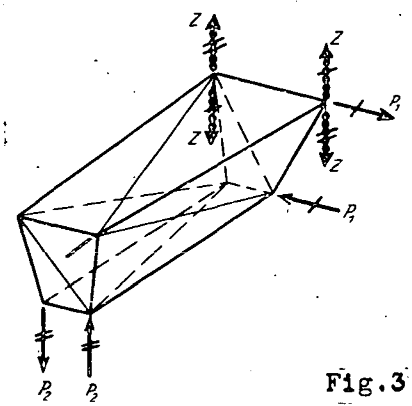


Fig.3

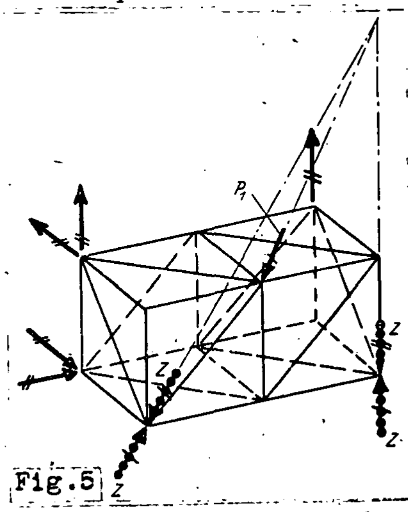


Fig.5

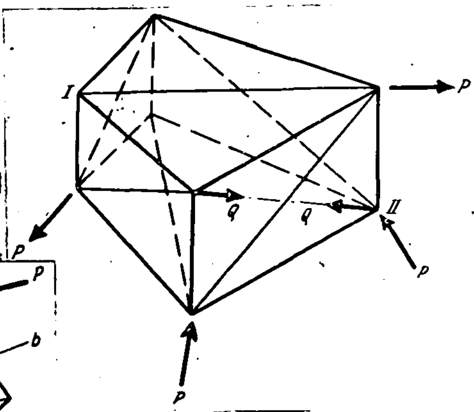


Fig.6

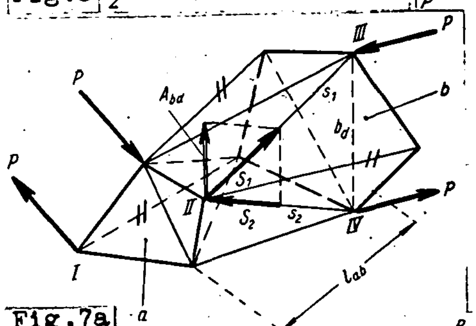


Fig.7a

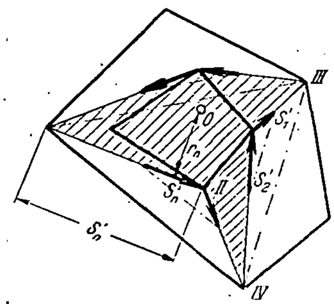


Fig.7b

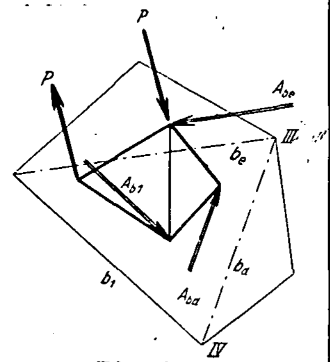
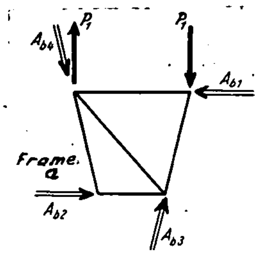


Fig.7c



$$A_{b\,m} = \mu_{a\,b} \cdot b_m$$

Fig.8c

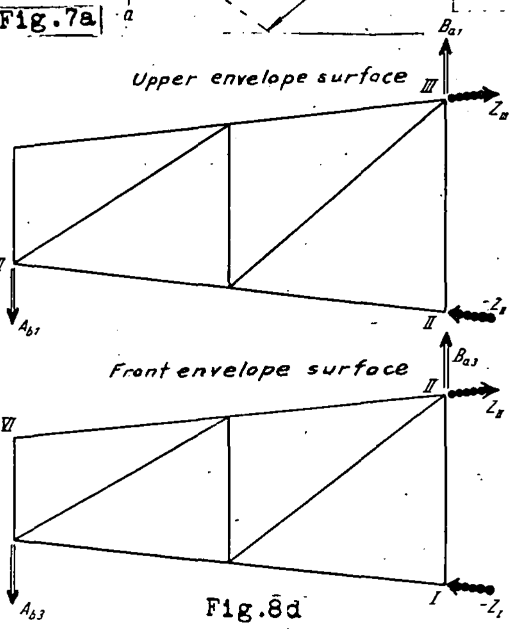
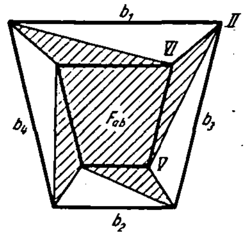


Fig.8d



$$\mu_{a\,b} = \frac{P_2 \cdot b_1}{2\,F_{a\,b}}$$

Fig.8b

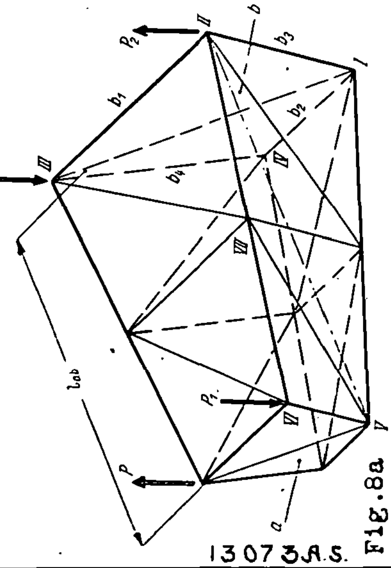


Fig.8a

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